5-1-1942

The Velocity of Falling Particles in a Liquid Medium

Elliott Coldwater

Coldwater, Elliott

The
Velocity of Falling Particles
In a Liquid Medium

by
Elliott Coldwater
Butte, Montana

A Thesis,

Submitted to the Department of
Mineral Dressing in Partial
Fulfillment of the Requirements
for the Degree of Bachelor of
Science in Metallurgical Engineering

Montana School of Mines
Butte, Montana
May 1, 1942

The
Velocity of Falling Particles
In a Liquid Medium

by

Elliott Coldwater
Butte, Montana

A Thesis,

Submitted to the Department of
Mineral Dressing in Partial
Fulfillment of the Requirements
for the Degree of Bachelor of
Science in Metallurgical Engineering

Montana School of Mines
Butte, Montana
May 1, 1942

Contents

THE VELOCITY OF FALLING PARTICLES IN A LIQUID MEDIUM

Introduction

The object of this paper is to present the results of an investigation of the velocity of free-falling mineral particles in a liquid medium.

The investigation was prompted by the fact that there seemed to be some discrepancy between the results published by Richards[1] and the empirical figures used in classification and in gravity concentration. Further, the work of Richards was restricted to quartz and galena whereas this study also includes pyrite and calcite.

Velocities computed from Rubey's equation[2] are compared with Richards' values and with the results of this work.

Acknowledgements

The writer wishes to express his gratitude to Dr. S. R. B. Cooke, who supervised the work, and whose aid in conducting the investigation was invaluable. Mention is due Armand Frederickson, who contributed many valuable suggestions, and Frank Casey, who aided in the construction of the equipment.

History of the Problem

In 1907, Robert H. Richards supervised an extensive study of the free-falling velocities of quartz and galena particles in water. G. A. Barnaby and Ralph Hayden performed the experimental work on the particle size ranging from 2.49 mm. to 0.28 mm.

The particles were first carefully sized by the use of sieves of the Tyler double series, then the average size of the particles in each sample obtained was determined by a microscope and micrometer scale, adopting the mean of a number of observations.

The grains in each size range were then dropped one by one through a one meter course in a vertical tube of water. One-half meter was used for the finer sizes. The elapsed time for the passage of each particle through the course was carefully measured and the results for each size range were averaged.

E. S. Bardwell investigated the size range from 0.48 mm. to 0.03 mm. by the use of the standard decantation method and then carefully measuring the grain size in each decanted fraction using a microscope and micrometer scale.

In both size ranges the grains were very carefully wetted to exclude the possibility of air bubbles adhering to the particles and then affecting the results.

In 1933 W. W. Rubey[2] published a purely mathematical formula for the determination of the velocity of particles falling in water. The formula is based upon Stokes' law of viscous resistance and the impact formula.

This formula contains no empirical constants and rather closely approximates the results obtained by experimental methods. However, the differences between Rubey's formula and experimental results are not constant throughout the range studied by Richards and the writer, and hence that formula is not practicable for use except for the most general approximations.

It should be pointed out that both Richards' work and Rubey's equation take into consideration only closely controlled laboratory samples and conditions.

In the present investigation an attempt was made to determine the average rate of fall of a sized group of particles of four minerals, quartz, pyrite, galena, and calcite.

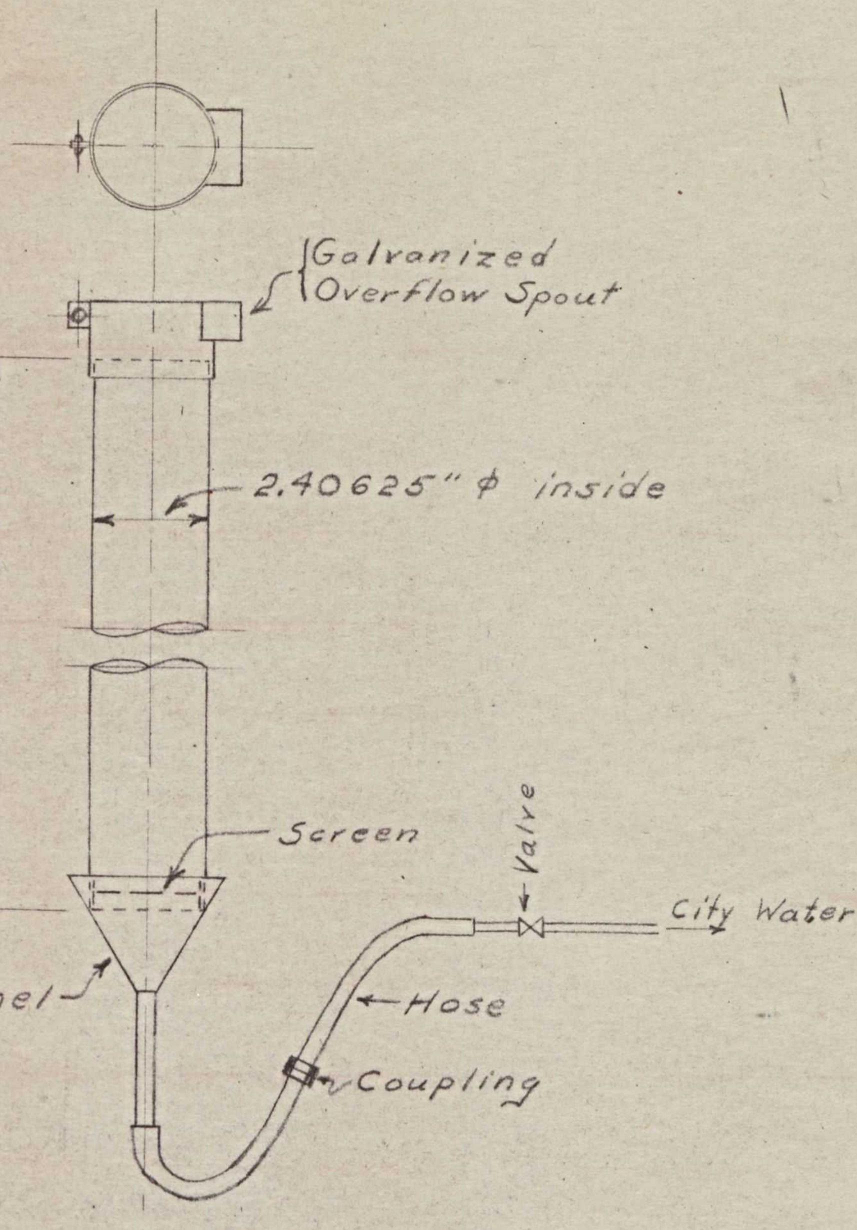
Method

The samples of pure quartz, galena, pyrite, and calcite were ground in a disc grinder until all of the material passed an eight-mesh Tyler screen. The samples then were very carefully sized by the use of the Tyler Double Series sieves. In this way a sample of each size range was obtained.

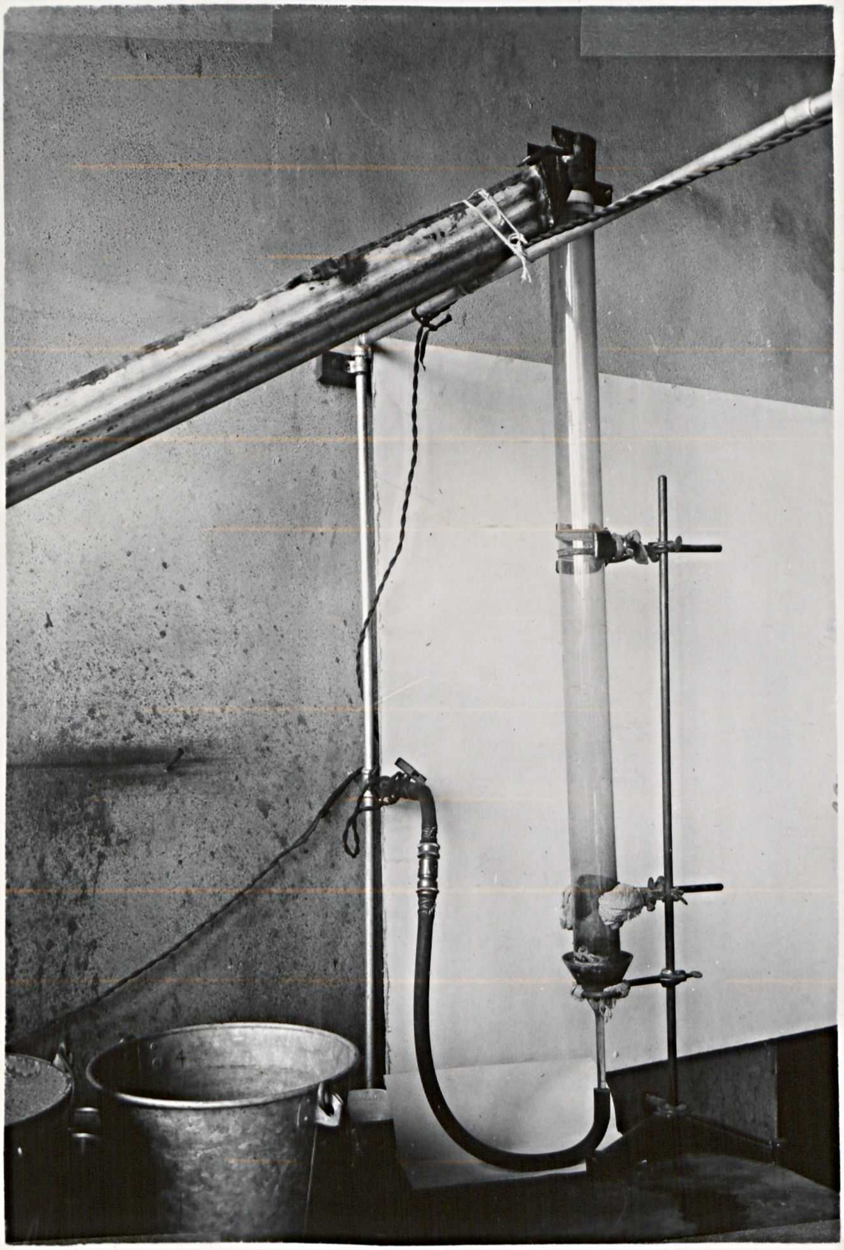
Rather than drop individual grains through a vertical course, it was decided to determine the velocity of a rising cureent of water necessary to keep a sample of each size range in teeter.

For this purpose a small constriction plate classifier was built. The glass classifier tube had a cross-sectional area of 29.3387 square centimeters. A twenty-mesh copper screen was used for the plate.

Water was introduced through a funnel at the bottom of the tube from a rubber hose and pipeline connected to the


Galvanized
Overflow Spout
2.40625" ϕ inside
42"
Screen
Valve
City Water
Funnel
Hose
Coupling

Classifier



Classifier

city water supply. Figure I shows the apparatus. A valve was placed near the end of the pipe-line and a snap coupling was inserted midway along the hose. A launder was used to carry the water from the top of the apparatus down to working level.

The principle of operation was to place a layer of the sized particles, about two grains deep, on the plate and then to admit water. The valve was very carefully adjusted until the rising current of water just kept the sample in teeter.

The overflowing water was collected in buckets and the weight of water flowing in a specified period of time was determined by weighing the filled buckets and subtracting the tare weight. The weight of the water was assumed to be one gram per cubic centimeter and this permitted conversion from weight to volume. The total volume for any one run was divided by the time in minutes. Using the formula $Q = AV$--in which Q is the volume per minute, A is the cross-sectional area of the tube, and V is the velocity of the water--the velocity of the rising current was calculated. In all cases V was converted to mm. per second.

The advantage of this method is that it readily gives the average falling velocity of a particular size group. This eliminates the necessity for computing a mean average velocity as required by previous methods.

Comparison of Velocity-Size Curves

The log sizes of the particles were plotted against the corresponding log velocities to compare the results of the various methods described.

Quartz

Using the theoretical Rubey curve as a base line, it was found that Richards' velocities were higher for the extemities of the size group studied and lower near the middle of the group.

The curve based upon the writer's results indicated slower velocities among the larger particles and much faster velocities in the small size range than either the Richards or Rubey curves.

All three curves are roughly parallel.

Galena

The Richards velocities are consistently higher than those given by Rubey while the velocities determined by the writer roughly follow the Rubey curve for the larger sizes but the plot of the rate of fall of the smaller sizes more nearly follows the Richards curve.

Pyrite

The graph based on the writer's results shows a consistently higher velocity for pyrite than the Rubey calculations indicate. However, the difference between velocities decreases as the larger size groups are approached.

Calcite

The writer's results shows that the velocities for calcite are definitely slower for the larger sizes and faster for the smaller sizes than those found from the Rubey equation.

Comparison of P-Q Curves

The Reynold's Number, P, and the Newtonian resistance factor, Q, were calculated for each velocity. Then numbers were plotted logarithmically.

Quartz

The Richards curve is parallel with and somewhat higher than the Rubey curve. The writer's curve is parallel with but higher than either Richards or Rubey for high Reynolds' numbers and also parallel to but below Rubey for small Reynolds' numbers. The offset in the writer's curve occurs where all three curves begin to level off.

Galena

The Richards curve is parallel to but much lower than the Rubey curve. The Writer's curve follows the Rubey curve for larger values of P but roughly follows the Richards curve when the value of Q begins to increase.

Pyrite

The writer's curve does not agree with the Rubey curve. Again a definite offset appears in the writer's curve.

Calcite

The writer's curve parallels the Rubey curve but it is higher for the larger Reynolds' numbers and lower for the smaller numbers with the offset again occurring at about the leveling off point.

Conclusions

The results obtained indicate rather clearly that some discrepancies do exist between the published data and the writer's experimental figures.

The short time available for research and the relatively small amount of work done make it impossible to establish a new set of standards in this report, but the figures obtained show that the problem is worthy of more investigation.

The establishment of a practical set of standards for the free-falling velocities of mineral particles would be of inestimable value in mineral dressing.

Rubey's Equation

$$V = \frac{\sqrt{\frac{20}{3} \cdot g \cdot \rho_F (\rho_P - \rho_F) d^3 + 360{,}000 \eta^2} - 600\eta}{\rho_F \cdot d}$$

Where:

v = settling velocity of particle in mm. per sec.
d = diameter of particle in mm.
ρ_P = density of particle
ρ_F = " " fluid
η = coef. of viscosity of fluid
g = acceleration due to gravity

Quartz: $\rho_P = 2.65$; $\rho_F = 1.00$; $\eta = 0.010$

$$V = \frac{\sqrt{10{,}791\, d^3 + 36} - 6}{d}$$

Galena: $\rho_P = 7.5$

$$V = \frac{\sqrt{42{,}510\, d^3 + 36} - 6}{d}$$

Pyrite: $\rho_P = 5.0$

$$V = \frac{\sqrt{26{,}160 d^3 + 36} - 6}{d}$$

Calcite: $\rho_P = 2.71$

$$V = \frac{\sqrt{11{,}183 d^3 + 36} - 6}{d}$$

RESULTS

QUARTZ

Screen Size	Time in Minutes	Net	Q	Velocity In mm/sec.
8				
9	2½	6388	2555	146.8
10	3	6732	2244	127.5
12	2½	4917	1966	115.0
14	2½	4516	1806	102.6
16	3	5516	1839	104.5
20	3	4440	1380	76.4
24	3	3851	1284	72.9
28	3	2796	932	52.9
32	2	1801	900.4	51.15
35	2	1620	809.9	46.01
42	2	1509	754.3	22.85
48	3	2255	785.1	44.6
60	1	627	627.	35.63
65	1	558	558.	31.71
80	2	988	493.8	28.05
100	2	851	425.5	24.17
115	2	417	208.6	11.85
150	2	403	201.4	11.44
170	3	595	198.4	11.27
200	4	475	118.8	6.75

QUARTZ

On Screen Mesh	Minimum Size In mm.	Rubey Velocity In mm/sec.	Coldwater Velocity In mm/sec.	Richards Grain Size In mm.	Richards Velocity In mm/sec.
8	2.380	158.2		2.44	168.
9	2.000	144.0	146.8	2.28	166.7
10	1.680	131.0	127.5	1.85	146.6
12	1.410	119.2	115.0	1.55	126.6
14	1.190	108.4	102.6	1.37	118.4
16	1.000	98.3	104.5	1.19	105.6
20	0.840	88.3	76.4	1.04	94.5
24	0.710	79.4	72.9	0.91	84.1
28	0.590	70.4	52.9	0.76	76.7
32	0.500	62.4	51.15	0.63	67.2
35	0.420	54.5	46.01	0.51	52.7
42	0.350	46.7	42.85	0.41	41.2
48	0.297	39.9	44.60	0.32	31.9
60	0.250	33.2	35.63	0.369	41.67
65	0.210	26.9	31.71	0.305	34.48
80	0.177	21.4	28.05	0.234	28.57
100	0.149	16.6	24.17	0.199	24.39
115	0.125	12.5	11.85	0.182	20.41
150	0.105	9.14	11.44	0.156	17.24
170	0.088	6.705	11.27	0.135	14.49
200	0.074	4.73	6.75	0.126	12.05
				0.121	10.20
				0.116	8.55
				0.112	7.14
				0.0912	6.02
				0.0846	5.05
				0.0800	4.26
				0.0747	3.57

TABLE I


3.0
2.5
2.0
1.5
1.0
0.5
Log Velocity (mm./sec.)
8.5
9.0
9.5
0.0
0.5
Log Size
(mm)
Rubey
Richards
Coldwater

Quartz

RESULTS

GALENA

Screen Size	Time in Minutes	Net	Q	Velocity In mm/sec.
8				
9				
10	1	4920	4920	279.5
12	1	4198	4198	238.5
14	1	3746	3746	212.8
16	1	3448	3448	195.9
20	1½	4806	3204	183.2
24	2	5412	2706	153.7
28	2	5316	2658	151.0
32	2	4786	2393	135.9
35	2	4446	2223	126.3
42	1½	3582	2388	135.7
48	2	4257	2129	120.9
60	2½	4436	1774	100.8
65	2	2809	1405	79.8
80	2	3065	1532	87.0
100	3	3087	1029	56.5
115	3	2925	975	55.4
150	3	2127	709	40.3
170	4	2960	740	42.06
200	4	2279	570	32.4

GALENA

On Screen Mesh	Minimum Size In mm.	Rubey Velocity In mm/sec.	Coldwater Velocity In mm/sec.	Richards Grain Size In mm.	Richards Velocity In mm/sec.
8	2.380	316.2		2.44	420.0
9	2.000	288.7		2.28	442.0
10	1.680	263.5	279.5	1.85	370.0
12	1.410	240.5	238.5	1.55	330.5
14	1.190	220.0	212.8	1.37	295.1
16	1.000	200.0	195.9	1.19	270.1
20	0.840	181.8	183.2	1.04	252.5
24	0.710	165.6	153.7	0.91	227.5
28	0.590	148.6	151.0	0.76	207.8
32	0.500	134.0	135.9	0.63	192.8
35	0.420	120.5	126.3	0.51	160.4
42	0.350	106.0	135.7	0.41	126.1
48	0.297	94.0	120.9	0.32	103.1
60	0.250	81.8	100.8	0.345	125.0
65	0.210	69.3	79.8	0.279	111.1
80	0.177	59.3	87.0	0.215	88.5
100	0.149	49.0	56.5	0.160	74.6
115	0.125	39.3	55.4	0.127	62.9
150	0.105	30.95	40.3	0.106	52.6
170	0.088	23.55	42.06	0.0967	43.5
200	0.074	17.7	32.4	0.0839	37.0
				0.0798	31.3
				0.0714	26.3

TABLE II


3.0
2.5
2.0
1.5
1.0
0.5
Log Velocity (mm. per sec.)
Rubey
Richards
Coldwater
8.5
9.0
9.5
0.0
0.5
Log Size (mm.)

Galena

RESULTS

PYRITE

Screen Size	Time in Minutes	Net	Q	Velocity In mm/sec.
8				
9	1¼	6159	4927	279.89
10	1	5098	5098	289.61
12	1½	5484	3656	207.67
14	1	3584	3584	203.60
16	1½	4450	2967	168.55
20	2	5640	2820	160.20
24	2	4621	2310	131.23
28	2¼	4646	2065	117.31
32	1½	3308	2206	125.32
35	2¼	4688	2083	118.33
42	2	3938	1969	111.86
48	2	3453	1727	98.11
60	2	3273	1637	92.99
65	2	2893	1447	82.20
80	2¼	2741	1218	69.19
100	2	2176	1088	61.81
115	2	1962	981	55.73
150	2	1807	903	51.30
170	2	1534	767	43.57

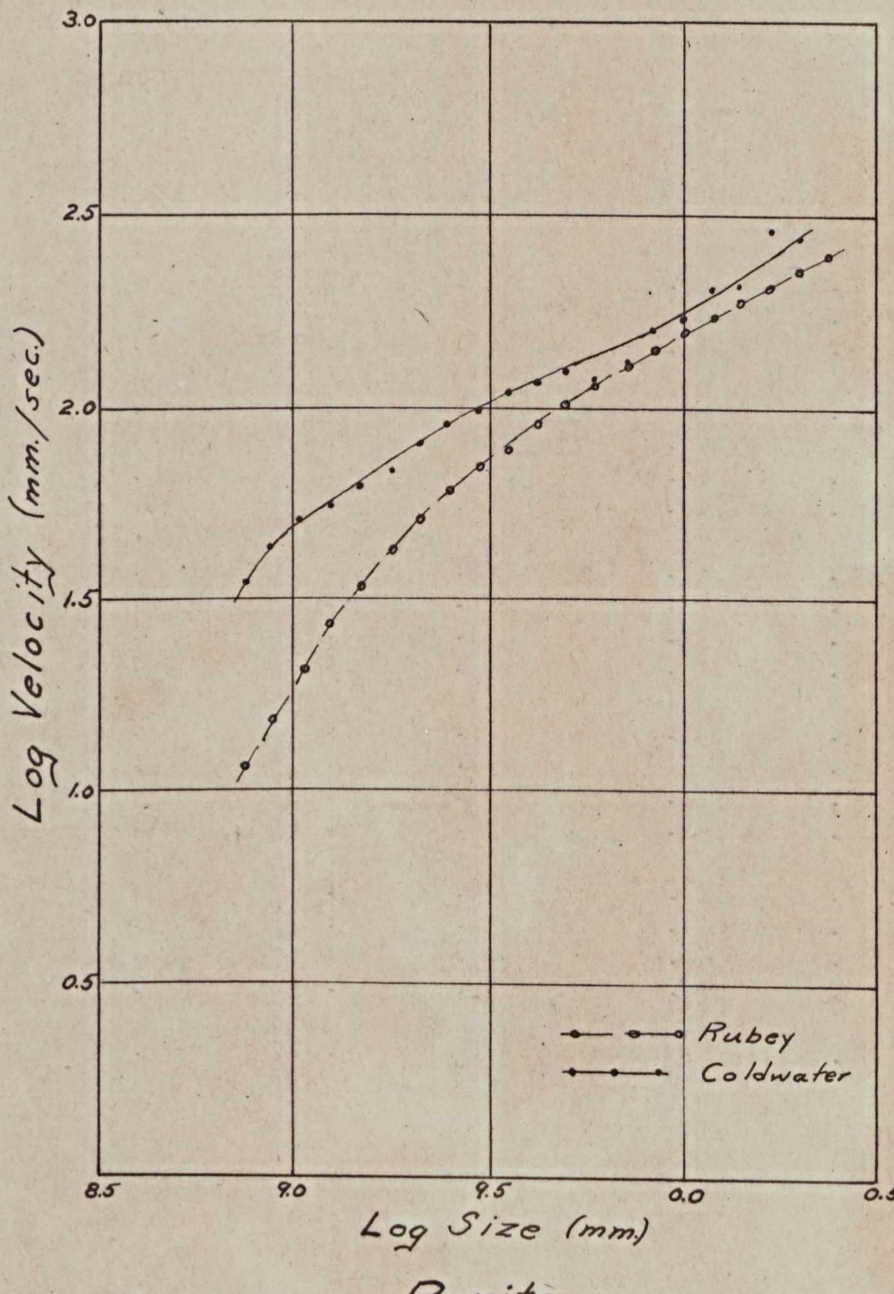
RESULTS

CALCITE

Screen Size	Time in Minutes	Net	Q	Velocity In mm/sec.
8				
9	2	4804	2402	136.5
10	2	3920	1960	115.0
12	2	3934	1967	111.8
14	2	3054	1527	86.77
16	2	2842	1421	80.75
20	2	2412	1206	68.58
24	2	2467	1233.5	70.07
28	2	2144	1072	60.90
32	$2\frac{1}{4}$	2126	945	53.70
35	2	1627	813.5	46.21
42	2	1547	773.5	43.94
48	2	1367	683.5	38.83
60	2	1692	846	46.06
65	2	1299	649.5	36.89
80	2	1091	545.5	30.99
100	2	883	441.5	25.08
115	2	660	330	18.75
150	2	494	247	16.03
170	2	276	138	7.84

On Screen Mesh	Minimum Size In mm.	Pyrite Velocity In mm/sec. Rubey	Pyrite Velocity In mm/sec. Coldwater	Calcite Velocity In mm/sec. Coldwater	Calcite Velocity In mm/sec. Rubey
8	2.380	247.0			161.0
9	2.000	225.5	279.89	136.5	146.5
10	1.680	206.0	289.61	115.0	133.7
12	1.410	188.0	207.67	118.0	121.0
14	1.190	171.5	203.60	86.77	110.6
16	1.000	156.0	168.55	80.75	100.1
20	0.840	141.0	160.20	68.58	90.0
24	0.710	128.4	131.23	70.07	81.0
28	0.590	114.5	117.31	60.90	71.7
32	0.500	103.0	125.32	53.70	63.6
35	0.420	91.5	118.33	46.21	55.7
42	0.350	77.2	111.86	43.94	47.7
48	0.297	70.3	98.11	38.83	40.8
60	0.250	60.4	92.99	46.06	34.0
65	0.210	50.8	82.20	36.89	27.6
80	0.177	42.2	69.19	30.99	21.05
100	0.149	34.1	61.81	25.08	17.1
115	0.125	26.8	55.73	18.75	12.8
150	0.105	20.5	51.30	16.03	9.52
170	0.088	15.1	43.57	7.84	6.94
200	0.074	11.35			5.00

TABLE III


3.0
2.5
2.0
1.5
1.0
0.5
Log Velocity (mm./sec.)
8.5
9.0
9.5
0.0
0.5
Log Size (mm.)
Rubey
Coldwater

Pyrite


3.0
2.5
2.0
1.5
1.0
0.5
Log Velocity (mm. per sec.)
8.5
9.0
9.5
0.0
0.5
Log Size (mm.)
Rubey
Coldwater

Calcite

Calculating Q & Reynold's Number "P"

$$Q = \frac{8r\,(\Delta - \Delta')g}{3v^2\Delta'}$$

where: r = radius of particle in cm.

Δ = specific gravity of particle

Δ' = " " " fluid = 1

g = acceleration due to gravity = 981 cm/sec.^{2}

v = velocity of particle in cm./sec.

Quartz: $\Delta = 2.65$, d = diameter of particle in cm.

$$Q = \frac{2158.2\,d}{v^2}$$

Galena: $\Delta = 7.5$

$$Q = \frac{8502\,d}{v^2}$$

Pyrite: $\Delta = 5.0$

$$Q = \frac{5232\,d}{v^2}$$

Calcite: $\Delta = 2.71$

$$Q = \frac{2236.68\,d}{v^2}$$

$$P = \frac{2rv\Delta'}{\mu}$$

where: r = radius of particle in cm.

use $2r = d$ = diameter of particle in cm.

v = velocity of particle in cm./sec.

Δ' = specific gravity of fluid = 1

μ = viscosity of fluid = 0.01

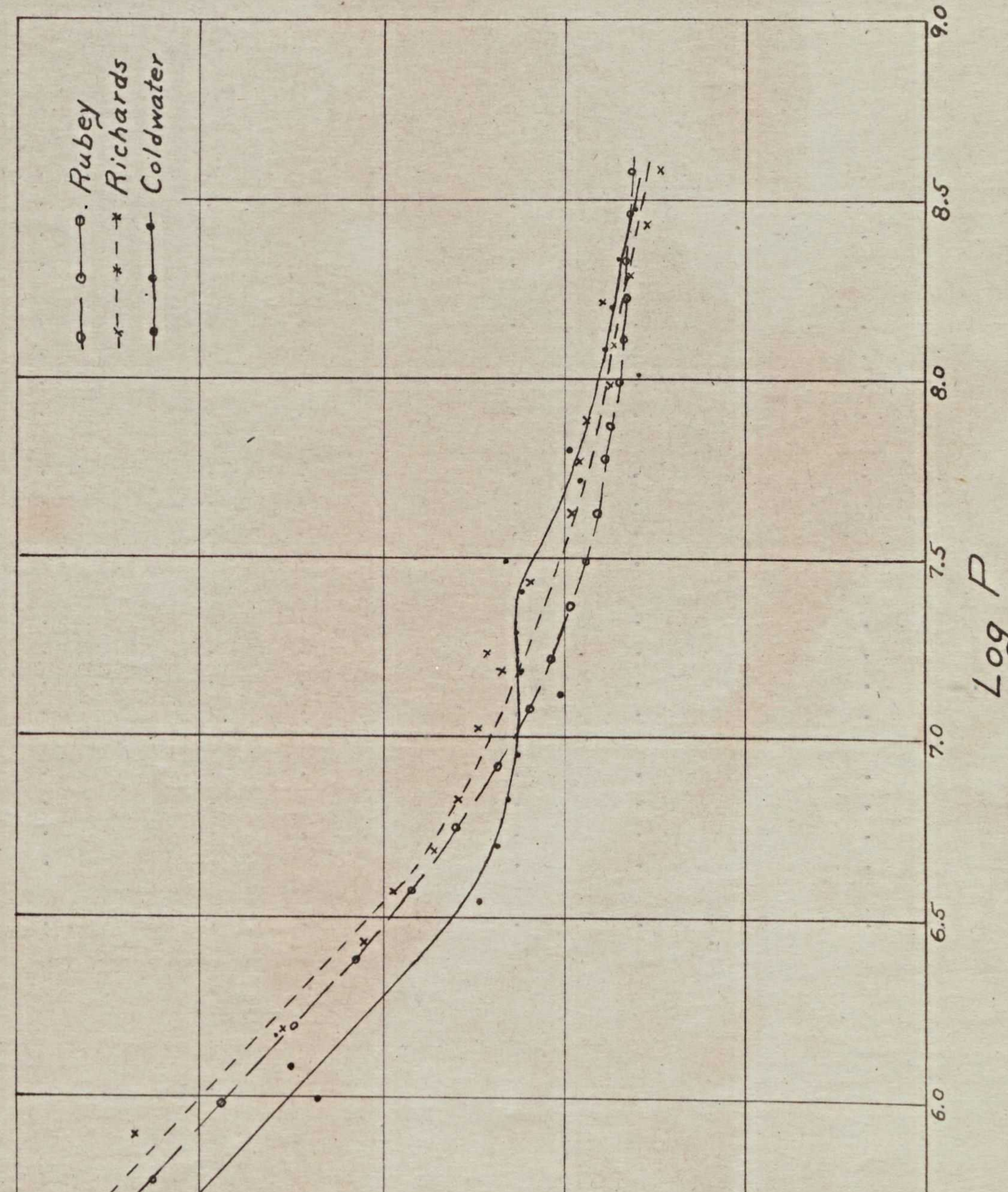
Therefore for all minerals

$$P = \frac{vd}{0.01}$$

QUARTZ

Mesh	P Rubey	Q Rubey	P Coldwater	Q Coldwater	P Richards	Q Richards
8	.03765	2.048			.038	1.77
9	.0288	2.055	.0294	2.002	.0271	1.857
10	.022	2.15	.0214	2.225	.0196	2.085
12	.0168	2.135	.0162	2.295	.01622	2.49
14	.01291	2.179	.0122	2.437	.01256	2.30
16	.00983	2.23	.01045	1.972	.00982	2.37
20	.00742	2.322	.00641	3.11	.00765	2.77
24	.00564	2.43	.00518	2.875	.00583	2.785
28	.00415	2.57	.00312	4.54	.00424	3.008
32	.00312	2.768	.00256	4.12	.00269	3.955
35	.00229	3.045	.00193	4.28	.00169	5.200
42	.001635	3.46	.00150	4.11	.00154	4.58
48	.001186	3.96	.001325	3.22	.001052	5.52
60	.000830	4.88	.000891	4.24	.000668	6.18
65	.000565	6.26	.000666	4.50	.000486	7.2
80	.000379	8.33	.000496	4.86	.000372	9.41
100	.000247	11.66	.000360	5.46	.000269	11.3
115	.0001563	17.24	.0001481	19.2	.000152	18.7
150	.0000960	27.12	.000121	17.82	.0000799	47.3
170	.0000590	42.2	.0000991	14.95	.0000427	71.5
200	.000035	71.3	.0000499	35.0	.00002665	126.2

TABLE IV

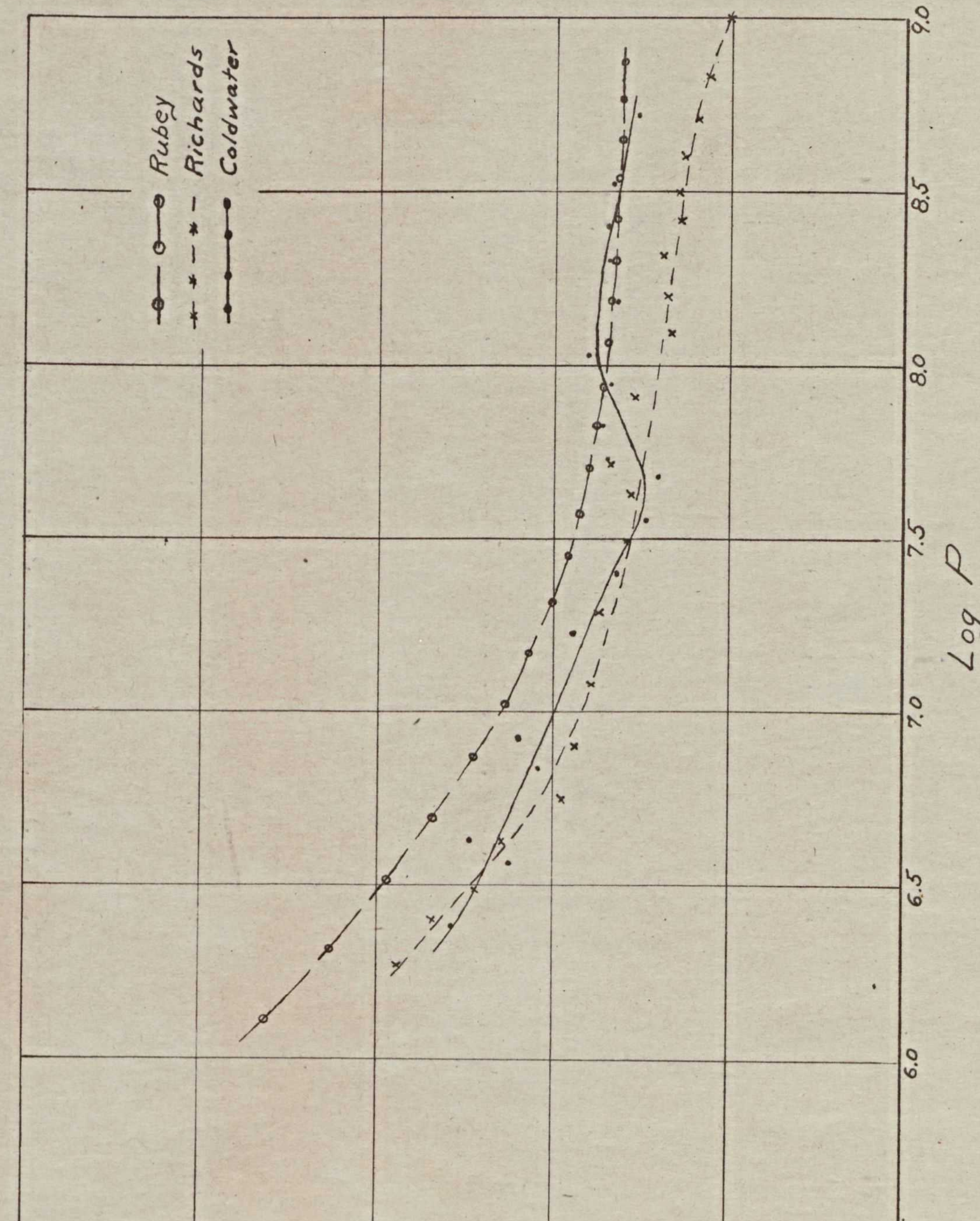

Rubey
Richards
Coldwater
Log Q
2.0
1.5
1.0
0.5
0.0
Log P
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0

Quartz

GALENA

Mesh	P Rubey	Q Rubey	P Coldwater	Q Coldwater	P Richards	Q Richards
8	.0754	2.025			.101	0.992
9	.0578	2.04			.0685	1.148
10	.0442	2.058	.0469	1.83	.0512	1.206
12	.0339	2.08	.0334	2.108	.0404	1.34
14	.0262	2.09	.02535	2.234	.03215	1.39
16	.0200	2.125	.01959	2.218	.02625	1.386
20	.01535	2.15	.0154	2.125	.0207	1.546
24	.01177	2.205	.0109	2.555	.0158	1.498
28	.00877	2.275	.00891	2.2	.01233	1.443
32	.00670	2.37	.00679	2.305	.00819	1.683
35	.00506	2.475	.0053	2.235	.00517	2.192
42	.00371	2.65	.00475	1.62	.00431	1.877
48	.00279	2.855	.003585	1.73	.00310	1.922
60	.002045	3.175	.00251	2.095	.001903	2.33
65	.001455	3.72	.00168	2.8	.001194	2.44
80	.00105	4.28	.00154	1.987	.000799	2.725
100	.000730	5.27	.000842	3.97	.0005575	2.96
115	.000492	6.88	.000693	3.47	.000421	4.34
150	.000325	9.32	.000423	5.49	.0003104	5.21
170	.000207	1 3.5	.000370	4.23	.000250	6.93
200	.000131	20.1	.000240	6.00	.0001877	8.78

TABLE V

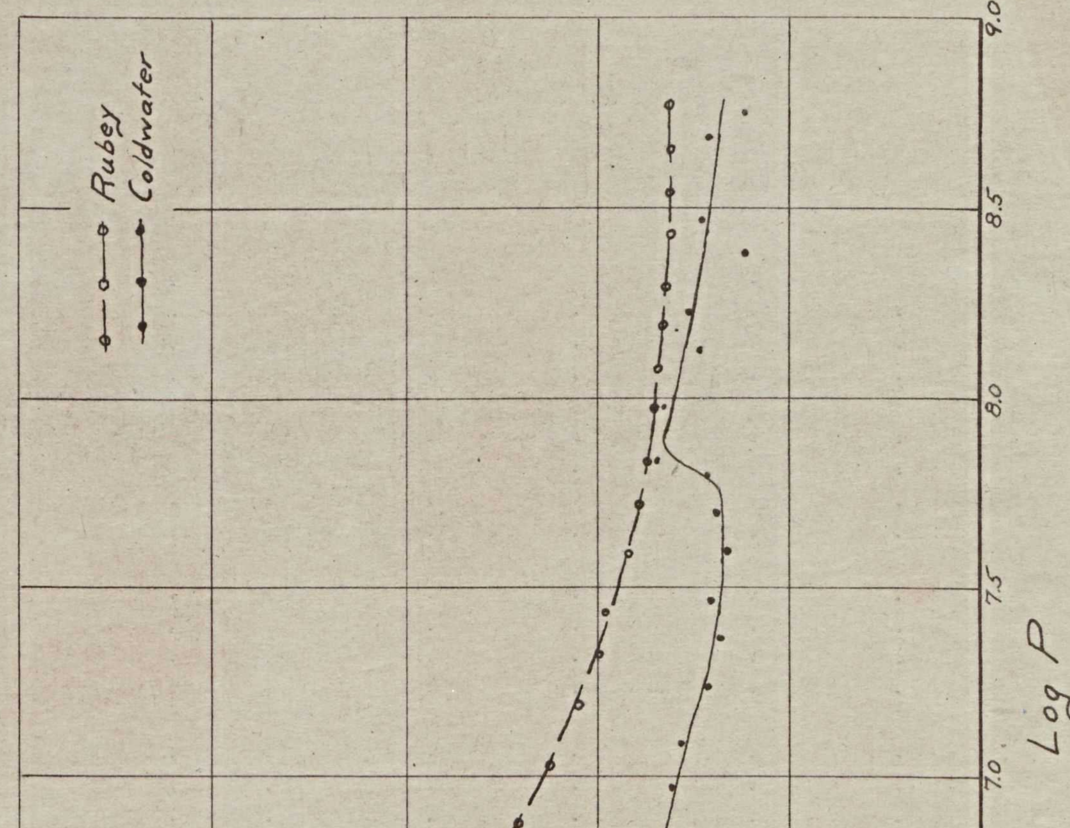

Rubey
Richards
Coldwater
Log P
Log Q
2.0
1.5
1.0
0.5
0.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0

Galena

PYRITE

On Screen Mesh	V Rubey In cm/sec.	P Rubey	V Coldwater In cm/sec.	P Coldwater	Q Rubey	Q Coldwater
8	24.7	.0588			2.081	
9	22.55	.0451	27.99	.05598	2.059	1.335
10	20.60	.0346	28.96	.0486	2.07	1.05
12	18.80	.0265	20.77	.02925	2.085	1.715
14	17.15	.0204	20.36	.0242	2.115	1.501
16	15.60	.0156	16.86	.01686	2.150	1.842
20	14.10	.01185	16.02	.01347	2.210	1.713
24	12.84	.00912	13.12	.00932	2.255	2.155
28	11.45	.00675	11.73	.00692	2.355	2.24
32	10.30	.00515	12.53	.00627	2.465	1.665
35	9.15	.00384	11.83	.00497	2.625	1.568
42	7.72	.00271	11.19	.00392	3.073	1.45
48	7.03	.00209	9.811	.002915	3.142	1.615
60	6.04	.00151	9.299	.002325	3.58	1.506
65	5.08	.001067	8.220	.00173	4.26	1.628
80	4.22	.000747	6.919	.001225	5.20	1.93
100	3.41	.000508	6.181	.000921	6.71	2.04
115	2.68	.000335	5.573	.000697	8.11	2.10
150	2.05	.000215	5.130	.000538	13.08	2.09
170	1.51	.000133	4.357	.000383	20.2	2.43
200	1.135	.000084			30.05	

TABLE VI

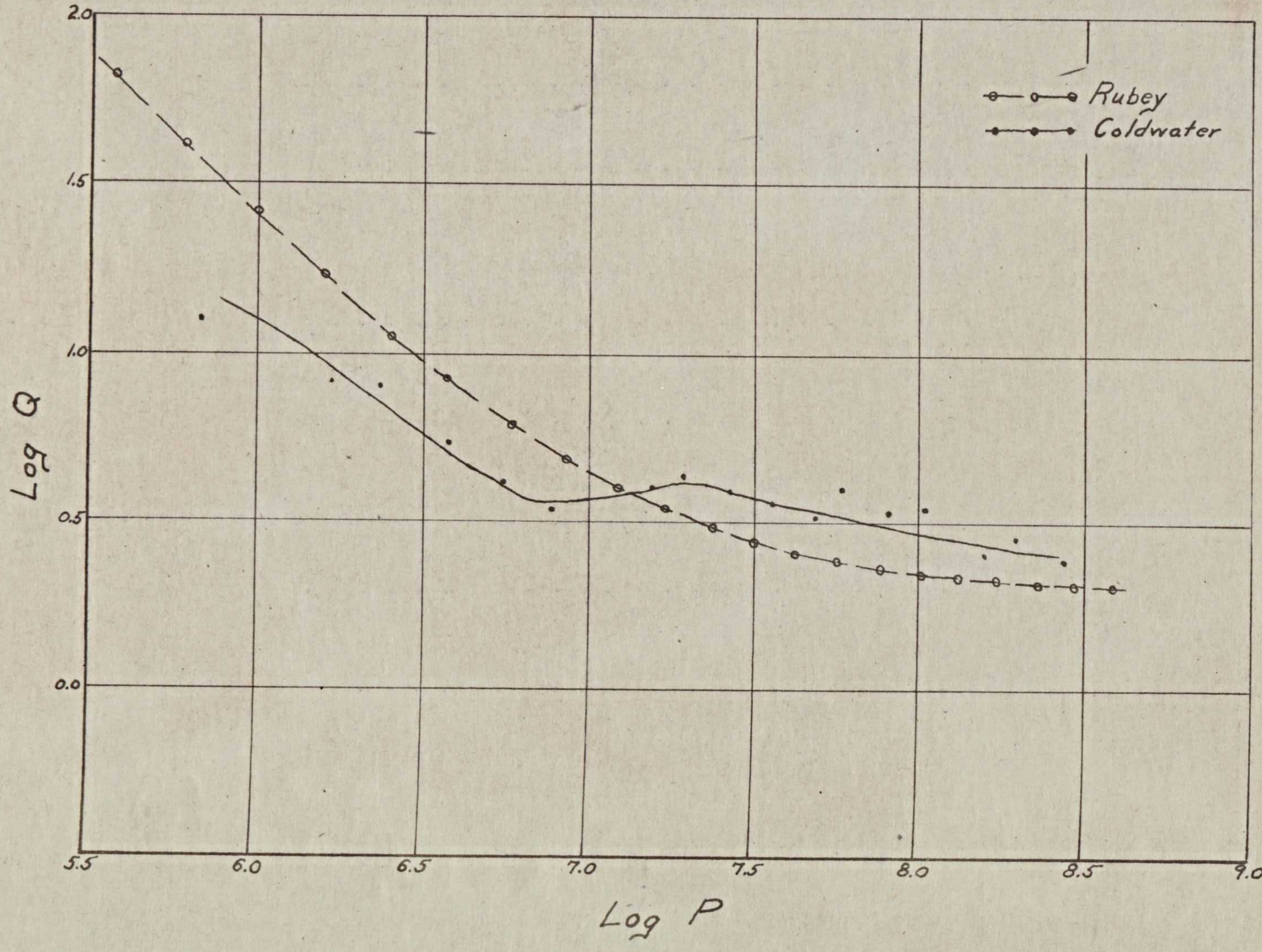


Pyrite

CALCITE

On Screen Mesh	V Rubey In cm/sec.	P Rubey	V Coldwater In cm/sec.	P Coldwater	Q Rubey	Q Coldwater
8	16.1	.0383			2.05	
9	14.65	.0293	13.65	.0273	2.08	2.405
10	13.37	.02245	11.50	.01932	2.10	2.84
12	12.10	.01705	11.18	.01575	2.15	2.52
14	11.06	.01315	8.667	.01031	2.175	3.54
16	10.01	.01001	8.075	.008075	2.23	3.43
20	9.0	.00756	6.858	.00576	2.317	3.98
24	8.1	.00575	7.007	.00497	2.415	3.24
28	7 .17	.00423	6.090	.00360	2.565	3.56
32	6.36	.00318	5.370	.002685	2.76	3.87
35	5.57	.00234	4.621	.00194	3.02	4.40
42	4.77	.00167	4.394	.00154	3.435	4.05
48	4.08	.00121	3.883	.001154	3.98	4.40
60	3.40	.000850	4.606	.00115	4.84	2.63
65	2.76	.000580	3.689	.000775	6.15	3.45
80	2.105	.000372	3.099	.000549	8.54	4.12
100	1.71	.0002545	2.508	.000374	10.38	5.51
115	1.28	.000160	1.875	.0002345	17.05	7.95
150	0.952	.000100	1.603	.000168	25.85	9.13
170	0.694	.0000611	0.784	.000069	40.8	32.0
200	0.500	.0000370			66.0	

TABLE VII



Calcite

References

1. Richards, Robert H. Velocity of Galena and Quartz Falling in Water. Trans. A.I.M.E. Vol. 38, (1907) pp. 210-235.

2. Rubey, William W. Settling Velocities of Gravel, Sand, and Silt Particles. Am. J. Sei. Series 5, Vol. 25, pp. 325-338.

31